GSE Geometry Milestone Study Guide Name _____ S: ____

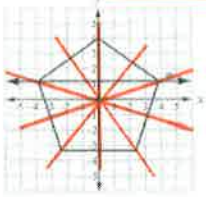
Unit 1 - Transformations

Isometry: A distance preserving map of a geometric figure to another location using a reflection, rotation or translation.

Rotation: Rules are in terms of counter clockwise $R_{90} = (-y, x)$ $R_{180} = (-x, -y)$ $R_{270} = (y, -x)$

Reflection: A transformation about a line that acts as a mirror; x = 0 is a vertical LoR & y = 0 is a horizontal LoR.

1) A regular pentagon is centered about the origin and has a vertex at (0, 4). Which transformation maps the pentagon onto itself?
A. a reflection across line m.
B. a reflection across the x-axis.
C. a clockwise rotation of 100° about the origin.
D. a clockwise rotation of 144° about the origin.

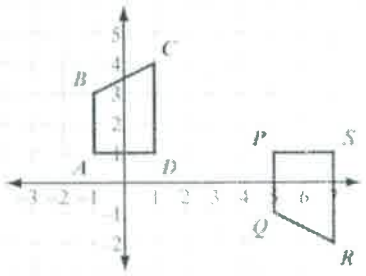


$\frac{360}{5} = 72°$

Any multiple of 72°

3) Describe transformations that map ABCD to PQRS.

many answers are accepted



translate right 6

Reflect over y=1

2) Is a dilation an isometry? Why?

No, does not produce a congruent shape

4) The point $Y(-1, 7)$ has been rotated 90° counter clockwise around the origin. Where is the new location of point Y?

$0°(x,y)$
$90(-y,x)$
$180(-x,-y)$
$270(y,-x)$

$(-1,7) \rightarrow (-y,x) \rightarrow Y' = (-7,-1)$

5) **Reflection:** About $y = 3$, gives what new vertices?
$H(12, -16), J(-9, -3), K(-17, 12), L(13, 11)$

sketch a picture!

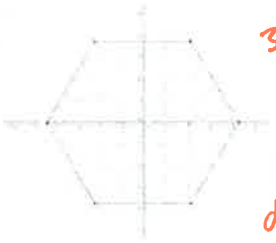
$H' =$ (12, 22)

$J' =$ (-9, 9)

$K' =$ (-17, -6)

$L' =$ (13, -5)

6) **Degrees of Rotation:** What is the minimum degrees of rotation to map the regular hexagon onto itself?

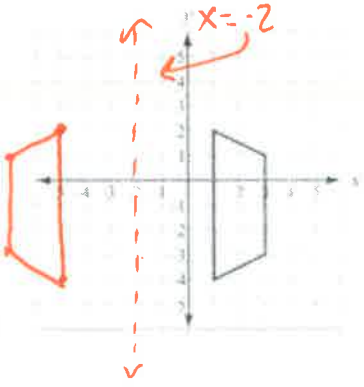


6 sides?

$\frac{360}{6} = 60°$

least of degrees is 60

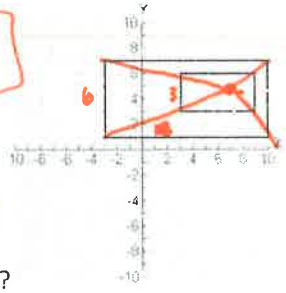
8) **Reflection:** About $x = -2$.



9) **Dilation:** A large rectangle is dilated to a smaller one. What is the scale factor & center of dilation?

new / old?

$\frac{3}{6} = \frac{1}{2} = k$



center (0,5) about

7) Give three multiples of rotations that map the hexagon onto itself?

60, 120, 180, 240, 300, 360

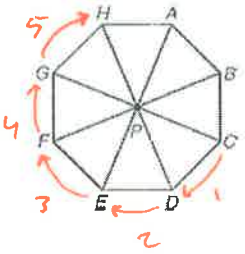
10) If the result of $(x, y) \rightarrow (x - 1, y + 2)$ *is* $A'(-5, 2)$, what is the **pre-image**, or A?

go backwards to find original

$(-5+1, 2-2)$

$A = (-4, 0)$

11) What ***clockwise*** rotation of the octagon at right about point P maps point C to point H?



8 sides so $\frac{360}{8} = 45°$

Each slice is 45°

C to H is 5 sections so $5 \cdot 45 = 225°$

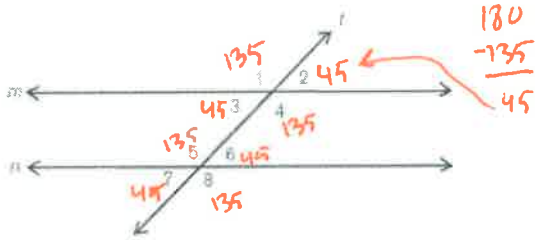
Unit 2 – Triangle Similarity & Congruence

1) Angle 5 is alternate interior to angle? ∠4

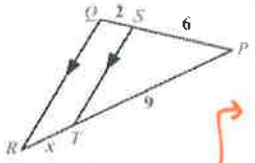
2) Angle 7 is corresponding to angle? ∠3

3) Angle 8 is vertical to angle? ∠5

4) If angle 1 equals 135 degrees, fill in all remaining angles.

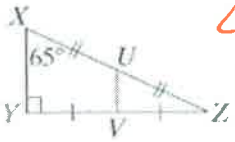


5) Triangle Proportionality: Find x.



$\frac{6}{9} = \frac{2}{x}$ → $6x = 18$ → $x = \frac{18}{6}$ → $x = 3$

6) Similar Triangles: What is the reflexive angle? Is XZY similar to UZV? If so, how?



∠Z is reflexive

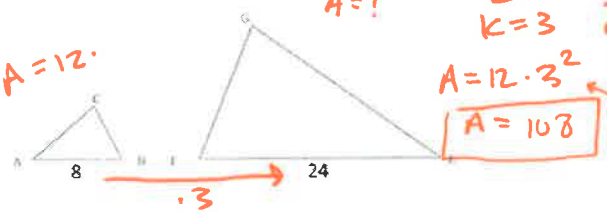
Yes by AA~

7) Midsegment: If M, N, and P are midpoints & perimeter of MPN = 64, find the length of all segments.

64 − 22 − 24 = 18

44

8) The sketch below shows 2 similar △'s, ABC and EFG. ABC has an area of 12 units, and it's base, AB, is 8 units long. The base of DEF is 24 units. What is the area of DEF?



A = 12 · A = ?

k = 3

Area is squared so scale factor is squared

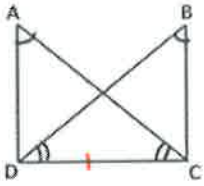
$A = 12 \cdot 3^2$

A = 108

·3

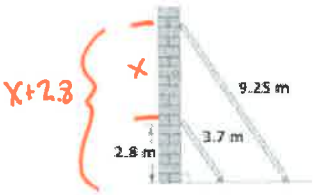
9) *Given:* $\measuredangle DAC \cong \measuredangle CBD, \measuredangle BDC \cong ACD$

Prove: $\overline{AC} \cong \overline{BD}$



Statement	Reason
1. ∠DAC ≅ ∠CBD	1. Given
2. ∠BDC ≅ ∠ACD	2. Given
3. CD ≅ CD	3. Reflexive Property
4. △CDA ≅ △DCB	4. AAS
5. AC ≅ BD	5. CPCTC

10) What is the height between the tops of the two ladders?



x + 2.8

$\frac{x+2.8}{2.8} = \frac{9.25}{3.7}$

$3.7x + 10.36 = 25.9$

$-10.36 \quad -10.36$

$3.7x = 15.54$

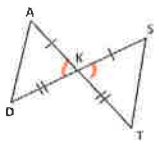
$\frac{3.7x}{3.7} = \frac{15.54}{3.7}$

x = 4.2

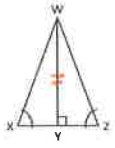
Triangle congruency: ***SSS, SAS, ASA, AAS, HL****. Remember SSA / ASS can't prove congruency.* ***<u>You can't double skip!</u>***

11) △RAC ≅ △RTO by ASA or AAS

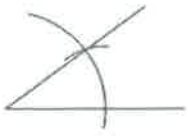
12) △KAD ≅ △KST by SAS



13) △XYW ≅ △ZYW by AAS



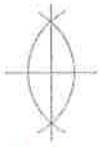
14) Geometric Constructions – Identify each partial or full construction



Copy Angle

bisect Angle

Parallel lines



Perpendicular bisector

Unit 3 - Right Triangle Trigonometry

Key Concepts

Finding missing sides use Soh Cah Toa

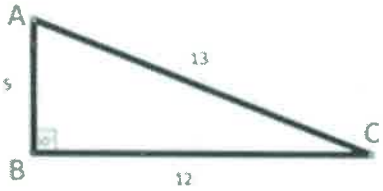
A missing side can be found using $\sin\theta = \left(\frac{o}{h}\right)$, $\cos\theta = \left(\frac{a}{h}\right)$, *or* $\tan\theta\left(\frac{o}{a}\right)$ *when you know an angle and one side of a right triangle.*

An angle θ *can be found by using one of* $\sin^{-1}\left(\frac{o}{h}\right)$, $\cos^{-1}\left(\frac{a}{h}\right)$, *or* $\tan^{-1}\left(\frac{o}{a}\right)$ *when two sides are known of a right triange.*

$Sin\ A = Cos\ B$ *when angles A and B are complementary in a right triangle:* $Sin\ A = Cos\ (90 - A)$

Using the diagram for 1-9. First, find each trig ratio.

1) Sin A = 12/13
2) Cos A = 5/13
3) Tan A = 12/5
4) Sin C = 5/13
5) Cos C = 12/13
6) Tan C = 5/12

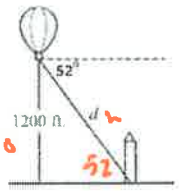


7) True or False: $Sin\ A = Cos\ (90 - A)$
Explain. True. because sinA = cosB if A & B are complements

8) Tan A & Tan B are Reciprocals

9) In a $45 - 45 - 90$ triangle, the ratio of Sin A = cosA because legs are congruent

10) **Angle of Depression & Elevation**: If the AoD is 52 degrees, solve for d.



$\sin(52) = \frac{1200}{x}$

$x = \frac{1200}{\sin(52)}$

$x = 152.82$

11) **Drop an altitude**: Solve for x.

$\sin(50) = \frac{11.15}{x}$

$x = \frac{11.15}{\sin(50)}$

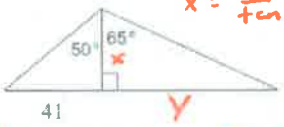
$x = 14.56$

$\sin(29) = \frac{y}{23}$

$y = 23 \cdot \sin(29)$

$y = 11.15$

12) **Area:** Solve for total area.



$\tan(50) = \frac{41}{x}$

$x = \frac{41}{\tan 50} \rightarrow 34.4$

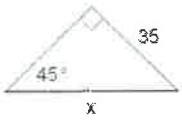
$\tan(65) = \frac{y}{34.4}$

$y = 34.4 \cdot \tan 65$

$y = 73.78$

$\frac{(41)(73.78) \cdot 34.4}{2} = 1974.4$

13) **Regular Trig: Find the missing side.**

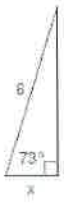


$\sin(45) = \frac{35}{x}$

$x = \frac{35}{\sin(45)}$

$x = 49.5$

14) **Regular Trig: Side**

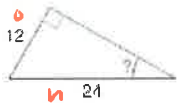


$\cos(73) = \frac{x}{6}$

$x = 6 \cdot \cos(73)$

$x = 1.75$

15) **Inverse Trig:** Find the angle.



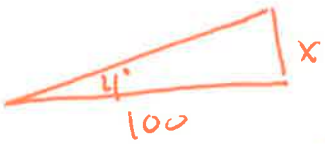
$\sin(?) = \frac{12}{24}$

$\sin^{-1}\left(\frac{12}{24}\right) = ?$

$? = 30°$

16) A road ascends a hill at an angle of 4° for every 100 feet of road, how many feet does the road ascend? **Draw a diagram.**

"goes up"



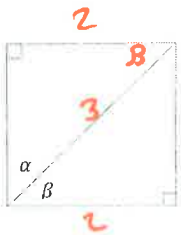
$\tan(4) = \frac{x}{100}$

$x = 100 \cdot \tan(4)$

$x = 6.99$ ft

17) In this figure, two right angles and two adjacent angles, α & β, are shown. If $\sin(\alpha) = \frac{2}{3}$, what is the value of $\cos(\beta)$?

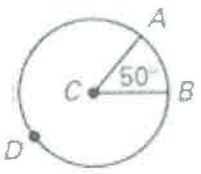
$\sin(\alpha) = \frac{2}{3} : \frac{opp}{hyp}$



$\cos B = \frac{adj}{hyp} = \frac{2}{3}$

Unit 4 – Circles, Angles & Segments

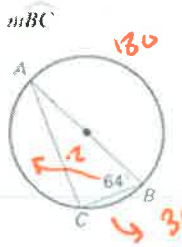
Central ∡ = Intercepted arc



AB = ? Central

$\overset{\frown}{AB} = 50$

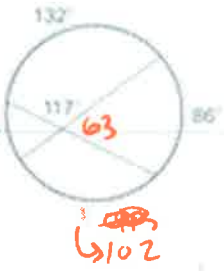
Diameter of Inscribed ∡



128 → 360 − 180 − 128 = 52°

Interior ∡ = ½ · (Barc + Larc) / 2

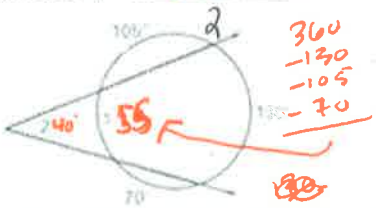
Find arcs 1 & 2.



40 = ?

↳ 102

Exterior ∡ = ½ (Barc − Larc)

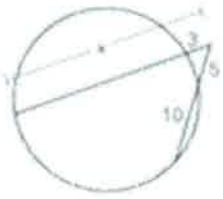


360 − 130 − 105 − 70 = 55°

Find arc 1 & angle 2/

$\frac{130-55}{2} = \angle 2$ → ∠2 = 37.5°

Outside(whole) = Out (Whole)

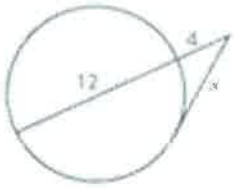


Find x. $3(x) = 5(15)$

$3x = 75$

$x = 25$

Tangent is outside & whole

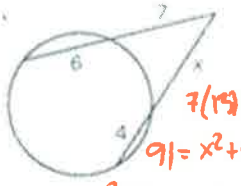


Find x. $4(16) = x(x)$

$64 = x^2$

$x = 8$

Missing Outside = Quadratic



$7(13) = x(x+4)$

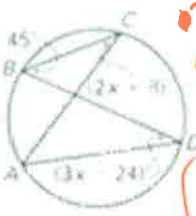
$91 = x^2 + 4x$

Find x. $x^2 + 4x - 91 = 0$

Use quad formula

$x = 7.75$

Shared Inscribed ∡'s are ≅



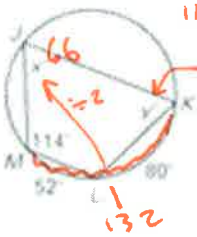
$2x + 8 = 3x - 24$

$32 = x$

$\angle BCA = 72°$

Find x & angle BCA.

Inscribed Quad is Supplementary



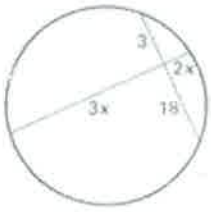
$114 + y = 180$

$y = 66$

$\angle L = 114$

Find x & y.

Product of Parts



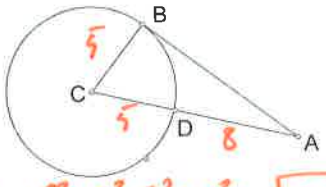
Find x and lengths of chords.

$3(18) = 3x \cdot (2x)$

$54 = 6x$ → $x = 9$

Point of Tangency: AB is tangent to circle C at B.

AD=8, CB=5, AB = ?



$13^2 - 5^2 = x^2$

$144 = x^2$ → $x = 12$

Arc Length & Sector Area

An apple pie has a diameter of 9 in. The pie is cut into 6 equal pieces. What is the area and arc length of 4 pieces of pie?

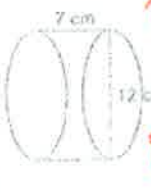
$\frac{4}{6} \cdot \pi \cdot 4.5^2$

$AL = 42.41 in$

Sphere Volume:

$\frac{4\pi r^3}{3} \rightarrow \frac{4\pi \cdot 6.4^3}{3} \rightarrow 1098.07$

Cylinder Volume:



$\pi r^2 h \rightarrow \pi \cdot 6^2 \cdot 7$

→ 16.16

RT Triangle Pyramid volume:

$LW \cdot h$

$\frac{3 \cdot 4 \cdot 6}{6} = 12$

Cavalieri's Principle: Can the cylinder and RTΔ pyramid at left have the same volume if they have the same height?

No Cross sectional Areas are not the same

Cone Volume:

$\frac{\pi r^2 h}{3}$

$\frac{\pi \cdot 12^2 \cdot 24}{3}$

3619.11

Composite Volume:

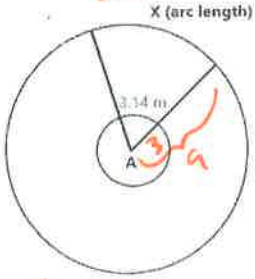
$\frac{4}{3}\pi r^3 / 2 = 452.4$

$\pi \cdot 6^2 \cdot 13 = 1470.3$

1922.7

At right, the radius of the smaller circle = 3 m while the radius of the larger circle is 9 m. The arc length intercepted by the small circle is 3.14 m. what is the arc length of the larger circle? s = AL, r = radius

$\frac{s}{r} = \frac{s}{r}$



$\frac{3}{3.14} = \frac{9}{x}$ → $3x = 28.26$

$x = \frac{28.26}{3} = 9.42$

Identify 2D shapes as 3D Objects: If a circle is rotated, what 3D shape will result?

Sphere

Unit 5 – Algebraic Connections with Geometry

Key Concepts

Distance: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, and you can always draw a right triangle on a graph to find Δx *and* Δy.

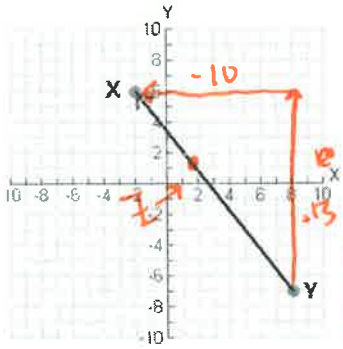
Midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Point Partitioning a Line Segment: $(x, y) = (x_1 + \frac{A}{A+B}(\Delta x), y_1 + \frac{A}{A+B}(\Delta y))$

Standard Form of a Circle: $(x - h)^2 + (y - h)^2 = r^2$, **where the number on the right is ALWAYS squared.**

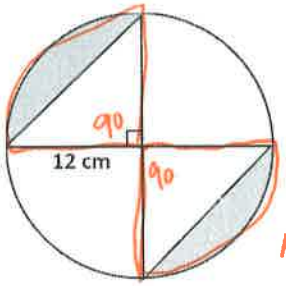
A parallelogram and rhombus have diagonals that bisect. A rectangle and square have diagonals that are congruent.

1) **Partitioning:** Find Point Z that partitions the directed line segment $\overline{YX}$ in a ratio of $\frac{5}{3}$, $X(-2, 6)$ *and* $Y(8, -7)$. Graph.



ΔX ΔY
5/8 [-10, +13]
x₁ y₁
(6.25, -7 + 8.25)
(1.75, 1.25)

2) **Sector Area:** 2 diagonals of a circle are shown, and the radius is 12 cm. What is the area of the shaded regions?



SA = arc/360 · πr²

SA = 90/360 · π · 12² = 72π = 226.19

Area of △ = b·h/2 = 12·12/2 = 72·2 = 144

So sectors minus Δ's

226.19 − 144 = 82.19

3) **Completing the Square:** Put into standard form, find center & radius. $4x^2 + 4y^2 - 24x + 48y + 13 = 0$

÷4

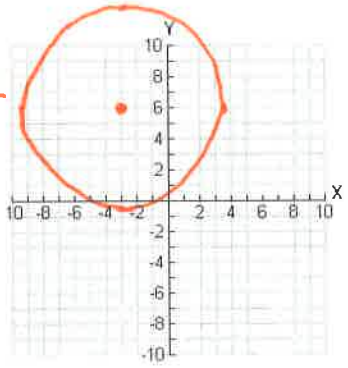
$x^2 - 6x + \_\_ + y^2 + 12y + \_\_ = -3.25 + \_\_ + \_\_$

$\frac{-6}{2} = -3^2 = 9$ ↑ $\frac{12}{2} = 6^2 = 36$ ↑

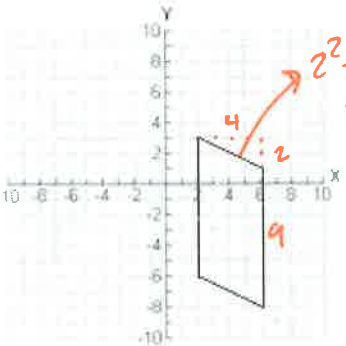
$(x-3)^2 + (y+6)^2 = 41.75$

$C = (3, -6)$ $r = \sqrt{41.75} = 6.5$

4) **Graphing Circles:** Now graph the circle from #3.



5) **Distance Formula:** Find the perimeter and area.



$2^2 + 4^2 = x^2$

$20 = x^2$

$x = 4.5$

$P = 9 + 9 + 4.5 + 4.5 = 27$

$A = 9 \cdot 4 = 36$

6) **Circle Properties:** Which point shown below lies on a circle with a center of $(3, -9)$ and a radius of $\sqrt{34}$?

$(6, -3)$ or $(1, -2)$ or $(1, -4)$ or $(0, -4)$

Either make equation and plug in points or use distance to match radius

$(x-3)^2 + (y+9)^2 = 34$

$(0-3)^2 + (-4+9)^2 = 34$ ✓

7) Find the midpoint: $(-10, -5)$ & $(13, 8)$

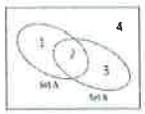
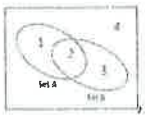
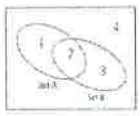
$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

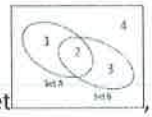
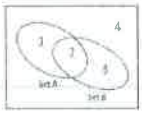
$\left(\frac{-10+13}{2}, \frac{-5+8}{2}\right)$

$\left(\frac{3}{2}, \frac{3}{2}\right) \rightarrow (1.5, 1.5)$

Unit 6 - Probability

Key Concepts

Given $A \cup B$ shade the set  , *Given* $A \cap B$ shade the set  , *Given* $\bar{A}$ *or* A' shade the set 

Given $(A \cup B)'$ shade the set  , *Given* $(A \cap B)'$ shade the set 

Addition Rule (aka mutually exclusive): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Multiplication Rule for Independent Events: $P(A \cap B) = P(A) * P(B)$

Conditional Probability: $P(A \cap B) = P(A) * P(B/A)$ **or** $P(B|A) = \frac{P(A \cap B)}{P(A)}$

Independent Events do not affect one another while Dependent Events do and means non-replacement.

1) Find the probability that a randomly selected student will be a junior, given that the student owns a car.

$P(J|OC) = \frac{P(J \cap OC)}{P(OC)} = \frac{6}{18} = \frac{1}{3} = .33$

2) Find the probability that a randomly selected student will own a car, given that the student is a senior.

$P(OC|S) = \frac{P(OC \cap S)}{P(S)} = \frac{12}{20} = \frac{3}{5} = .6$

Car Ownership by Grade

	Owns a Car	Does Not Own a Car	
Junior	6	10	16
Senior	12	8	20
TOTAL	18	18	

3) For two events B and C, it is known that $P(C|B) = 0.65$ and $P(C \cap B) = .43$. Find P(B).

$P(C \cap B) = P(B) \cdot P(C|B) \rightarrow .43 = ? \cdot .65$

$? = \frac{.43}{.65} \rightarrow .66 = P(B)$

4) For two events X and Y, it is known that $P(X) = \frac{5}{24}$ and $P(X \cap Y) = \frac{1}{8}$. Find $P(Y|X)$.

$P(Y|X) = \frac{(X \cap Y)}{(X)} \rightarrow \frac{\frac{1}{8}}{\frac{5}{24}} \rightarrow \frac{1}{8} \cdot \frac{24}{5} = \frac{3}{5} = .6$

4) A sock drawer contains 5 pairs of each color socks: white, green and blue. What is the probability of randomly selecting a pair of blue socks, replacing it, and then randomly selecting a pair of white socks?

$P(B) \cdot P(W)$

$\frac{5}{15} \cdot \frac{5}{15} = \frac{25}{225} = \frac{1}{9} = .11$

5) Randy has 8 pennies, 3 nickels, and 5 dimes in his pocket. If he randomly chooses 2 coins, what is the probability that they are both pennies if he doesn't replace the first one?

$P(P) \cdot P(P|P)$

$\frac{8}{16} \cdot \frac{7}{15} = \frac{56}{240} = \frac{7}{30} = .23$

6) Using the letters in the state MISSISSIPPI. Find the probability of picking an **S** and then a **P** without replacement.

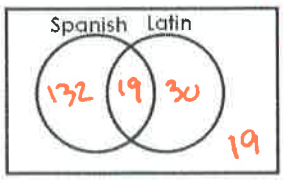
$\frac{4}{11} \cdot \frac{2}{10} = \frac{8}{110} = \frac{4}{55} = .07$

7) Determine if the following events are independent.

$P(A) = \frac{3}{4}, P(B) = \frac{5}{6}, P(A \cap B) = \frac{5}{8}$.

$\frac{3}{4} \cdot \frac{5}{6} = \frac{5}{8} \rightarrow \frac{5}{8} = \frac{5}{8}$ so yes

A guidance counselor is planning schedules for 200 students. 151 want to take Spanish and 49 want to take Latin. 19 say they want to take both. Display this information on the Venn Diagram.



151 − 19 = 132

49 − 19 = 30

8) What's the probability that a student studies at least one subject? $P(S \cup L) \rightarrow \frac{181}{200} = .91$

9) What's the probability that a student studies exactly one subject? $132 + 30 \rightarrow \frac{162}{200} = .81$

10) What's the probability that a student studies neither subject? $P(S \cup L)' \rightarrow \frac{19}{200} = .095 \rightarrow .10$

11) What's the probability that a student studied Spanish if it is known that he, she studies Latin? $P(S|L) = \frac{P(S \cap L)}{P(L)} = \frac{19}{49} = .39$

12) If you roll two die, find:

P(Odd number or a number greater than 8)

$\frac{6}{12} + \frac{0}{12} = \frac{6}{12} = \frac{1}{2} = .5$

13) If you roll two die, find:

P(Doubles or a sum of 6)

$\frac{6}{36} + \frac{5}{36} - \frac{1}{36}$ (overlap)

$\frac{10}{36} = \frac{5}{18} = .28$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12