Name MASTER S: ____

Unit 1 - Transformations

Key Concepts

Transformation: The mapping, or movement, of all points of a figure in a plane according to a common operation, such as translation, reflection or rotation.

Translation: A transformation that slides each point of a figure the same distance in the same direction.

Angle of Rotation: The amount of rotation (in degrees) of a figure about a fixed point such as the origin.

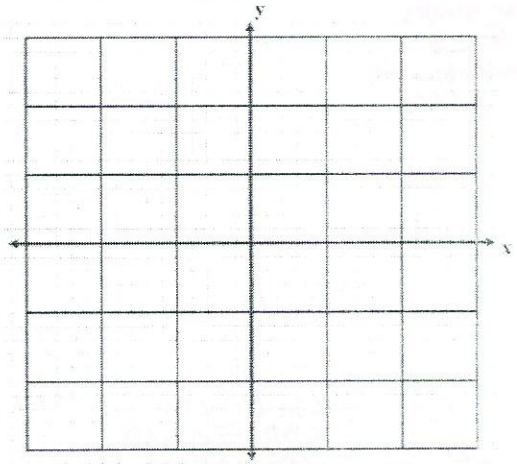
Rotation: A transformation that turns a figure about a fixed point through a given angle and a given direction, such as 90° which is understood to be counter clockwise. $R_{90} = (-y, x)$ $R_{180} = (-x, -y)$ $R_{270} = (y, -x)$

Reflection Line (or line of reflection): A line that acts as a mirror so that corresponding points are the same distance from the mirror. x = 0 is a vertical line of reflection, while y = 0 is a horizontal line of reflection

Isometry: A distance preserving map of a geometric figure to another location using a reflection, rotation or translation.

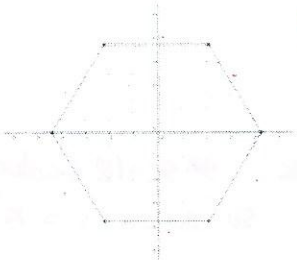
Even Symmetry means refect about $y - axis$ *or* $(-x, y)$ rule. **Odd Symmetry** means 180 rotation symmetry or $(-x, -y)$ rule.

Draw a rectangle, parallelogram, square, or isosceles trapezoid in the coordinate plane so that portions of the shape are in each of the four quadrants. Explain what would happen to your shape if you transformed it using each of the given rules.



1. $(-x, y)$ Reflect across the y-axis
2. $(x, -y)$ Reflect across x-axis
3. $(x + 3, y)$ Horizontal translation right 3 units
4. $(x, y - 2)$ Vertical shift down 2 units
5. $(x - 1, y + 4)$ shift left 1 unit & up 4 units
6. $(2x, 2y)$ dilation by a factor of 2.
7. $(-x, -y)$ Rotation 180°
8. $(3x + 2, y - 1)$ dilation

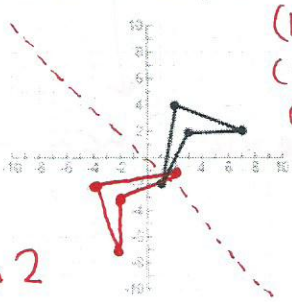
9) What is the minimum degrees of rotation to map the regular hexagon onto itself?



6 = order

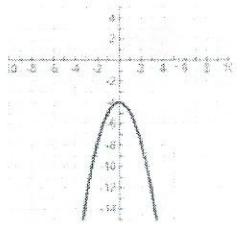
$\frac{360}{6} = 60°$

10) Reflect about the line $y = -x$.



(1, -2) → (2, -1
(3, 2) → (-2, -3
(2, 4) → (-4, -
(7, 2) → (-2, -

11) Is this function even, odd or neither?



even

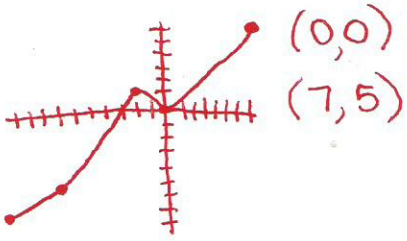
12) Are these equations even, odd or neither?

$f(x) = 3x^2 - 2x^1 + 1x^0$ neither

$f(x) = -9x^4 + 3x^2 + 2x^0$ even

13) A function is known to have odd symmetry. If its graph contains the following points in addition to many more, state at least two points that would also have to lie along the graph.

$(-10, -8), (-7, -5), (-2, 1)$



(0,0)
(7,5)

Unit 2 – Triangle Similarity & Congruence

Key Concepts

1) Vertical angles, alternate interior angles, alternate exterior angles and corresponding angles are congruent.

2) Shared or reflexive sides and angles are congruent.

3) Triangle Similarity: AA~, SSS~, and SAS~

4) A line drawn inside a triangle that is parallel to a 3rd side creates proportionality amongst corresponding parts.

5) A midsegment is 1/2 as long as the 3rd side that it's parallel to and bisects the sides it touches.

6) Triangle Congruency: SSS, SAS, ASA, AAS and HL. ASS or SSA cannot be used to prove congruency.

7) Constructions: look over your practice examples. Remember the key concept is measuring more than ½ way to make secondary arc strikes in some of the constructions.

1)

The sketch below shows two triangles, ΔABC and ΔEFG. ΔABC has an area of 12 square units, and its base (AB) is equal to 8 units. The base of ΔEFG is equal to 24 units.

a. How do you know that the triangles are similar? AA~

b. Name the pairs of corresponding sides and the pairs of corresponding angles. How are the corresponding sides related and how are the corresponding angles related? Why is this true?

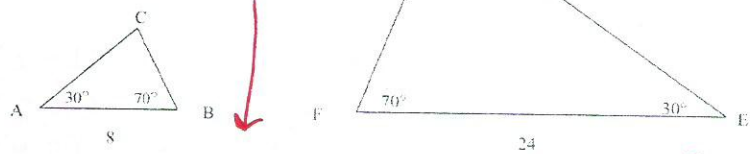
$\angle A \cong \angle E$ $\angle B \cong \angle F$ $\angle C \cong \angle G$ $AC \sim EG$ $CB \sim GF$ $AB \sim EF$ similar shapes have $\cong$ ∠'s and proportionate sides

c. What is the area of Δ EFG? Explain your reasoning.

d. What is the relationship between the area of ΔABC and the area of Δ EFG? What is the relationship between the scale factor and the ratio of the areas of the two triangles? Use an area formula to justify your answer algebraically.

$\frac{24}{8} = 3 = k$ scale factor is 3

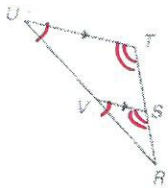
So Area is $= k^2$



if $a = 12$ then $\rightarrow a = 12 \cdot k^2 \rightarrow 12 \cdot 3^2$

108 units²

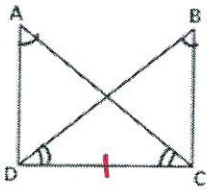
2) Are the two triangles similar? If so, explain how you know based on angle types.



yes by AA~

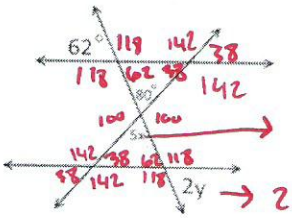
3) *Given:* $\angle DAC \cong \angle CBD$, $\angle BDC \cong \angle ACD$

Prove: $\overline{AC} \cong \overline{BD}$



Statement	Reason
1. $\angle DAC \cong \angle CBD$	1. Given
2. $\angle BDC \cong \angle ACD$	2. Given
3. $CD = CD$	3. Reflexive Property
4. $\Delta CDA \cong \Delta DCB$	4. AAS
5. $AC \cong BD$	5. CPCTC

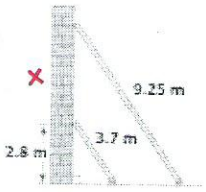
4) Find ALL missing angles.



$5x = 80 \rightarrow x = \frac{80}{5} = 16$

$2y = 62 \rightarrow y = \frac{62}{2} = 31$

5) What is the height between the tops of the two ladders?



$\frac{2.8}{3.7} = \frac{2.8+x}{9.25}$

$25.9 = 10.36 + 3.7x$

$-10.36 \quad -10.36$

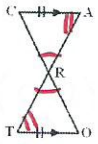
$15.54 = 3.7x$

$\frac{15.54}{3.7} = \frac{3.7x}{3.7}$

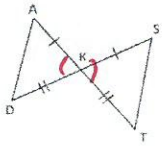
$4.2 = x$

Write the triangle congruency statement & by theorem, using SSS, SAS, ASA, AAS, HL or write "Not possible" if not congruent.

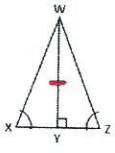
6) $\Delta RAC \cong \Delta RTO$ by AAS



7) $\Delta KAD \cong \Delta KTS$ by SAS



8) $\Delta XYW \cong \Delta ZYW$ by AAS



Unit 3 - Right Triangle Trigonometry

Key Concepts

Finding missing sides use Soh Cah Toa

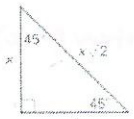
An angle θ can be found by using one of $\sin^{-1}\left(\frac{o}{h}\right), \cos^{-1}\left(\frac{a}{h}\right), or \tan^{-1}\left(\frac{o}{a}\right)$ *when two sides are known of a right triange.*

Sin A = Cos B when angles A and B for the complementary in a right triangle.

THEOREM 5.1: 45°-45°-90° TRIANGLE THEOREM

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

hypotenuse = leg · $\sqrt{2}$

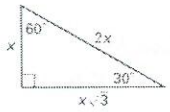


THEOREM 5.2: 30°-60°-90° TRIANGLE THEOREM

In a 30°-60°-90° triangle, the hypotenuse is 2 as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

hypotenuse = 2 · shorter leg

longer leg = shorter leg · $\sqrt{3}$



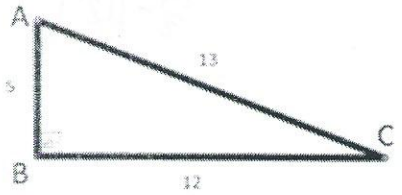
Using the diagram, Find each trig ratio.

Sin A = 12/13 *Sin C =* 5/13

Cos A = 5/13 *Cos C =* 12/13

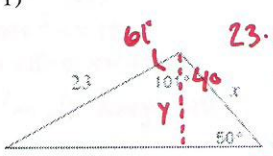
Tan A = 12/5 *Tan C =* 5/12

Are Sin A & Cos B the same?



Solve for x. (Multistep, drop an altitude)

1)



$\cos(61) = \frac{y}{23}$

$23 \cdot \cos(61) = y$

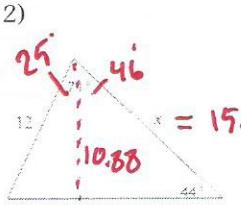
$y = 11.15$

$\sin(50) = \frac{11.15}{x}$

$x \cdot \sin(50) = 11.15 \rightarrow x = \frac{11.15}{\sin 50}$

$x = 14.56$

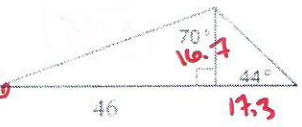
2)



$= 15.66$

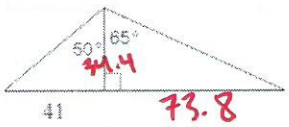
Solve for total area by finding the area of each smaller Δ.

3)



528.56

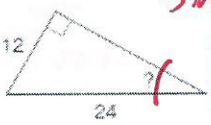
4)



1974.56

Find the missing angle (inverse trig).

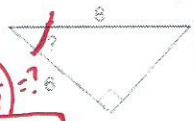
5)



$\sin^{-1}\left(\frac{12}{24}\right) = ?$

$? = 30°$

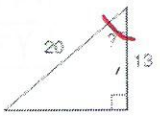
6)



$\cos^{-1}\left(\frac{6}{8}\right) = ?$

$? = 41.4°$

7)



$\cos^{-1}\left(\frac{13}{20}\right) = ?$

$? = 49.5°$

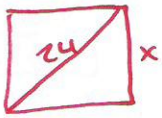
8) What is the length of the diagonal of a square with side lengths $3\sqrt{2}$?

$3\sqrt{2} \cdot \sqrt{2} = ?$

$3 \cdot 2$

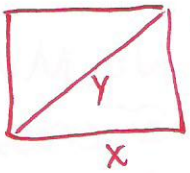
6

9) The length of the diagonal of a square is 24. What is the length of each side?



$x = \frac{24}{\sqrt{2}} \rightarrow x = 12\sqrt{2} \rightarrow 16.97$

11) The area of a square is 25 cm². What is the product of the lengths of the diagonals of the square?



$A = 25$

$x^2 = 25$

$x = 5$

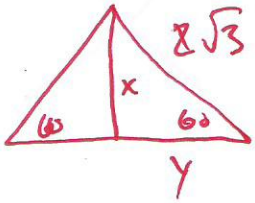
$y = 5\sqrt{2}$

$y \cdot y \rightarrow 5\sqrt{2} \cdot 5\sqrt{2}$

$25 \cdot 2$

50

12) What is the length of the altitude of an equilateral triangle with side lengths $8\sqrt{3}$?



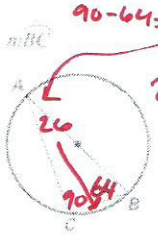
$y = \frac{8\sqrt{3}}{2} \rightarrow 4\sqrt{3}$

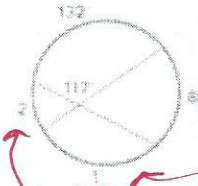
$x = 4\sqrt{3} \cdot \sqrt{3}$

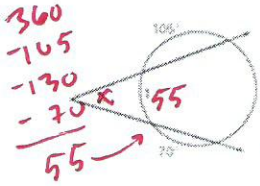
$x = 4 \cdot 3 \rightarrow 12$

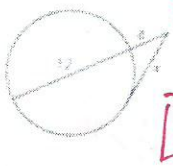
Key Concepts

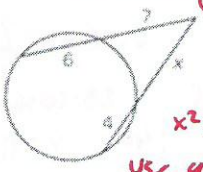
1) A central angle's relationship to its intercepted arc is that they are congruent. 2) An intercepted arc's relationship to its inscribed angle is that it is 2 times the measure of the angle. 3) A diameter with endpoints of an inscribed angle creates a right triangle with a 90 degree angle opposite the diameter. 4) Arc Length is based on the fraction of the arc measure over 360 x $2\pi r$. 5) Sector Area is based on the fraction of the arc measure over 360 x πr^2. 6) When finding an exterior angle of secants and, or tangents, take ½ of the difference of the arcs. 7) When finding an interior angle of secants and or chords, take ½ of the sum of the arcs. 8) When finding missing segments of secants or chords inside a circle, multiple the parts of the segments. 9) When finding missing segments of secants and, or tangents, multiply the outside • (whole). 10) Tangents that intersect outside a circle are congruent. 11) Opposite angles of quadrilaterals inscribed inside a circle are supplementary (=180). 12) A Point of tangency creates a right angle, making the tangent perpendicular to the radius. When solving problems, look for a 2nd radius being part of the length of the hypotenuse. 13) A radian is an angle measure such that the length of the radius equals the arc length. 14) Cavalieri's Principle says that the volume of two solids is equal if two parallel cross sections have the same area.

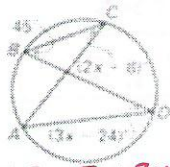










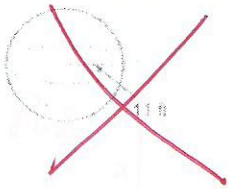


Find volume given diameter.

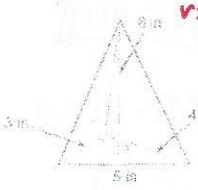
Find the volume.

Find the volume.

Find the surface area.



Find the volume.



Find the volume.

Find the arc length & sector area.

An Apple pie has a diameter of 9 in. The pie is cut into 6 equal pieces. What is the area of each piece of the pie?

$\frac{1}{6} \cdot 4.5^2 \cdot \pi = 10.6 \text{ in}^2$

Based on Cavalieri's Principle, can a rectangular prism have the same volume as a rectangular pyramid?

No Must have ≅ cross sections Area at equal heights.

Key Concepts

Distance: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, and you can always draw a right triangle on a graph to find Δx *and* Δy.

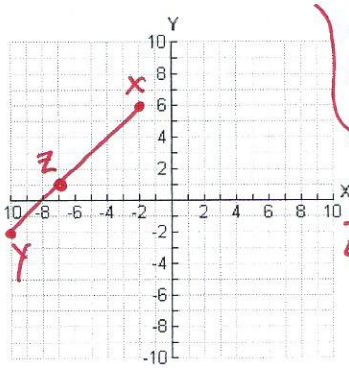
Midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Point Partitioning a Line Segment: $(x, y) = (x_1 + \frac{A}{A+B}(\Delta x), y_1 + \frac{A}{A+B}(\Delta y))$

Try to rotate a polygon like a triangle or parallelogram so an altitude is perpendicular to a side or base.

Circle Standard Form Equation: $(x - h)^2 + (y - h)^2 = r^2$, where the number on the right is ALWAYS squared.

1) Find Point Z that partitions the directed line segment XY in a ratio of $5:3, X(-2, 6)$ *and* $Y(-10, -2)$. Graph.



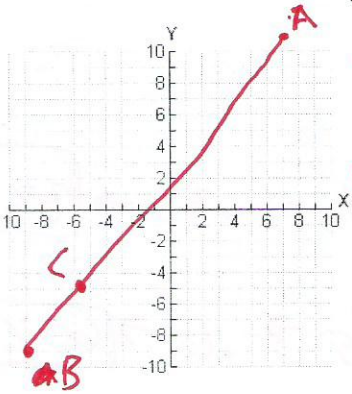
$\Delta x = (-10 + 2) = -8$

$\Delta y = (-2 - 6) = -8$

$\left(-2 + \frac{5}{8}(-8), 6 + \frac{5}{8}(-8)\right)$

$Z = (-7, 1)$

2) Partition the long segment from point $A(7, 11)$ *to* $B(-9, -9)$ *by* $\frac{4}{5}$. Graph.



$\Delta x = (-9 - 7) = -16$

$\Delta y = (-9 - 11) = -20$

$C = \left(7 + \frac{4}{5}(-16), 11 + \frac{4}{5}(-20)\right)$

$C = (-5.8, -5$

Put into standard form, find center & radius (simplest form).

$$\frac{4x^2 + 4y^2 - 24x + 48y + 13 = 0}{4}$$

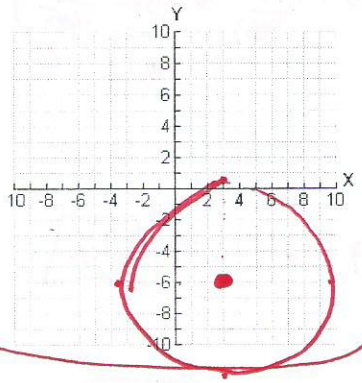
$x^2 + y^2 - 6x + 12y + 3.25 = 0$

$x^2 - 6x + \frac{9}{} + y^2 + 12y + \frac{36}{} = -3.25 + \frac{9}{} + \frac{36}{}$

$\frac{-6}{2} = -3^2 = 9$ $\quad \frac{12}{2} = 6^2 = 36$

$(x-3)^2 + (y+6)^2 = 41.75$

Now graph the circle.

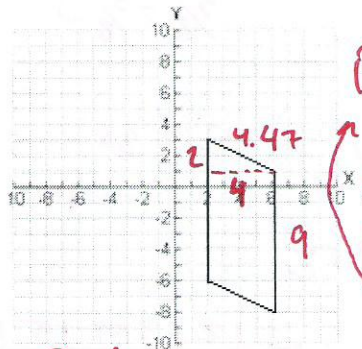


$C = (3, -6)$

$r = \sqrt{41.75}$

$r = 6.5$

Find the perimeter and area.



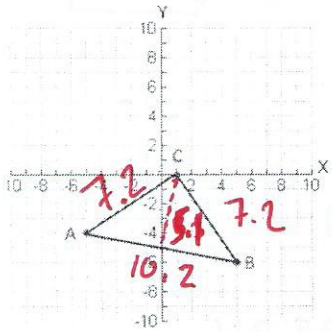
$P = 26.94$

$A = 36$

$P = (4.47 \cdot 2) + (9 \cdot 2) =$

$A = 9 \cdot 4 =$

Find the perimeter and area.



$P = 7.2 + 7.2 + 10.2$

$P = 24.6$

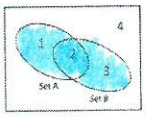
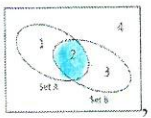
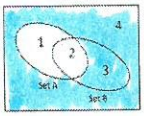
$A = \frac{7.2 \cdot 7.2}{2} = 25.9$

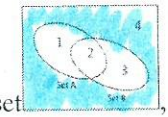
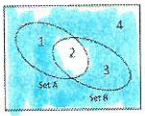
or

$A = \frac{10.2 \cdot 5.1}{2} = 26.0$ or

Unit 6 - Probability

Key Concepts

Given $A \cup B$ shade the set  , *Given* $A \cap B$ shade the set  , *Given* $\bar{A}$ *or* A' shade the set 

Given $(A \cup B)'$ shade the set  , *Given* $(A \cap B)'$ shade the set 

Addition Rule (aka mutually exclusive): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Multiplication Rule for Independent Events: $P(A \cap B) = P(A) * P(B)$

Conditional Probability: $P(A \cap B) = P(A) * P(B/A)$ or $P(B|A) = \frac{P(A \cap B)}{P(A)}$

Independent Events do not affect one another while Dependent Events do and means non-replacement.

1) Find the probability that a randomly selected student will be a junior, given that the student owns a car.

$6/18 \rightarrow 1/3 \rightarrow .33 \rightarrow$ 33.3%

2) Find the probability that a randomly selected student will own a car, given that the student is a senior.

$12/20 \rightarrow 3/5 \rightarrow .6 \rightarrow$ 60%

Car Ownership by Grade

	Owns a Car	Does Not Own a Car
Junior	6	10
Senior	12	8
TOTAL	18	18

3) For two events B and C, it is known that $P(C|B) = 0.65$ and $P(C \cap B) = .43$. Find P(B).

$P(C|B) = \frac{P(C \cap B)}{P(B)} \rightarrow .65 = \frac{.43}{P(B)} \rightarrow P(B) = \frac{.43}{.65}$

.661

4) For two events X and Y, it is known that $P(X) = \frac{5}{24}$ and $P(X \cap Y) = \frac{1}{8}$. Find $P(Y|X)$.

$P(Y|X) = 1/8 / 5/24 \rightarrow 1/8 \cdot 24/5 \rightarrow$ .6

4) A sock drawer contains 5 pairs of each color socks: white, green and blue. What is the probability of randomly selecting a pair of blue socks, replacing it, and then randomly selecting a pair of white socks?

$5/15 \cdot 5/15 = 1/9 = .111 \rightarrow$ 11.1%

P(b) P w

5) Randy has 8 pennies, 3 nickels, and 5 dimes in his pocket. If he randomly chooses 2 coins, what is the probability that they are both pennies if he doesn't replace the first one?

$\frac{8}{15} \cdot \frac{7}{14} \rightarrow \frac{7}{30} \rightarrow .233 \rightarrow$ 2.33%

P(P) P(P|P)

6) Using the letters in the state MISSISSIPPI. Find the probability of picking an S and then a P without replacement.

$\frac{4}{11} \cdot \frac{2}{10} = \frac{4}{55} = .073 \rightarrow$ 7.3%

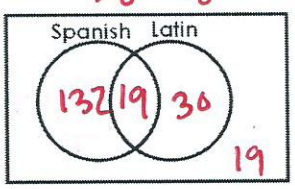
P(S)

7) Using $P(A \cap B) = P(A) * P(B)$, determine if the following events are independent.

$P(A) = \frac{3}{4}, P(B) = \frac{5}{6}, P(A \cap B) = \frac{5}{8}$.

$\frac{3}{4} \cdot \frac{5}{6} = \frac{5}{8}$ $\frac{5}{8} = \frac{5}{8}$ } yes independe

A guidance counselor is planning schedules for 200 students. 151 want to take Spanish and 49 want to take Latin. 19 say they want to take both. Display this information on the Venn Diagram.



8) What's the probability that a student studies at least one subject? P(SL) $181/200 \rightarrow$ 90.5%

9) What's the probability that a student studies exactly one subject? $162/200 \rightarrow$ 81%

10) What's the probability that a student studies neither subject? P(SL)' $19/200 \rightarrow$ 9.5%

11) What's the probability that a student studied Spanish if it is known that he, she studies Latin? $19/49 \rightarrow$ 38.8%

12) If you roll two die, find:

P(Odd number or a number greater than 8)

$\frac{18}{36} + \frac{10}{36} - \frac{6}{36} = \frac{22}{36} \rightarrow$ 61.1%

overlap

13) If you roll two die, find:

P(Doubles or a sum of 6)

$\frac{6}{36} + \frac{5}{36} - \frac{1}{36} =$ 27.8%

overlap

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12