 Name ______________________ S: ____

Unit 1 - Transformations

Isometry: A distance preserving map of a geometric figure to another location using a reflection, rotation or translation.

Rotation: Rules are in terms of counter clockwise $R_{90} = (-y, x)$ $R_{180} = (-x, -y)$ $R_{270} = (y, -x)$

Reflection: A transformation about a line that acts as a mirror; x = 0 is a vertical LoR & y = 0 is a horizontal LoR.

Transformations: Describe each transformation.

1. $(-x, y)$
Reflection over y-axis

2. $(x, -y)$
reflection over x-axis

3. $(x + 3, y)$
translate right 3

4. $(x, y - 2)$
translate down 2

5. $(x - 1, y + 4)$
Translate left 1 & up 4

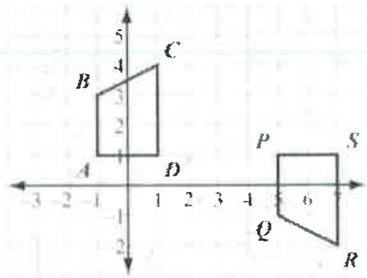
6. $(2x, 2y)$
dilate by factor of 2

7. $(-x, -y)$
Rotation 180°

8. $(3x + 2, y - 1)$
horizontal stretch by 3
translate right 2 & down 1

Is a dilation an isometry?

9) Describe transformations that map ABCD to PQRS.



10) The point $Y(-1, 7)$ has been rotated 90° counter clockwise around the origin. Where is the new location of point Y?

(cw
0°: (x,y)
90°: (-y,x)
180°: (-x,-y)
270°: (y,-x)
360°: (x,y)

Y: (-1,7) ⇒ $\boxed{Y' = (-7, -1)}$

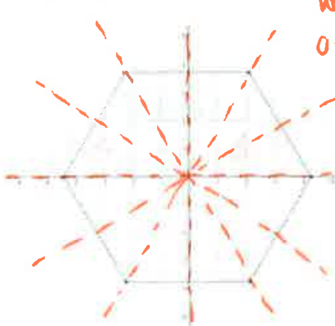
11) **Reflection:** About $y = 3$, gives what new vertices? $H(12, -16), J(-9, -3), K(-17, 12), L(13, 11)$

y=3

H'= (12, 22) J'= (-9, 9)

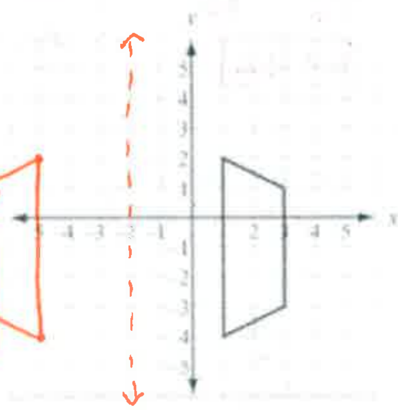
K'= (-17, -6) L'= (13, -5)

12) **Degrees of Rotation:** What is the minimum degrees of rotation to map the regular hexagon onto itself?



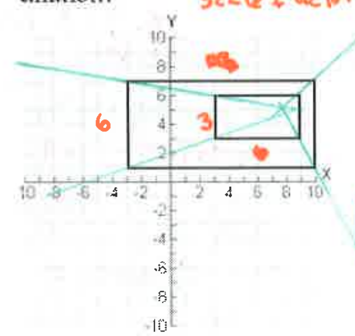
order=6
Mag= 360/6
$\boxed{60°}$

13) **Reflection:** About $x = -2$.



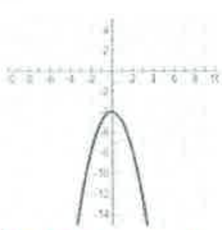
14) **Dilation:** A large rectangle is dilated to a smaller one. What is the scale factor & center of dilation?

Scale factor = new/old → 3/6 → $\boxed{K = 1/2}$



Symmetry Rules: The even y-axis symmetry rule is $(-x, y)$ and the odd 180 rotation symmetry rule is $(-x, -y)$.

15) **Visual:** Is this function even, odd or neither?



even → y-axis symmetry

16) **Exponents:** Are these equations even, odd or neither?

$f(x) = 3x^2 - 2x^1 + 1^0$
neither has both

$f(x) = -9x^4 + 3x^2 + 2^0$
Even has only Even Powers

17) **Rule-Based:** A function is known to have odd symmetry. If its graph contains the following points in addition to many more, state at least two points that would also have to lie along the graph.

$(-10, -8), (-7, -5), (-2, 1)$

odd has 180° symmetry and touches origin
so → (0,0) (10,8) (7,5) (2,-1)

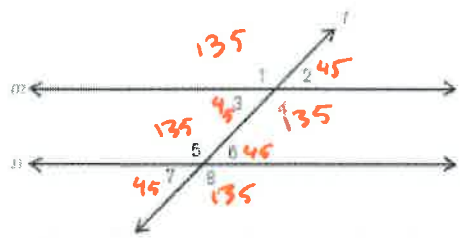
1) Angle 1 is alternate exterior to angle? ∠8

2) Angle 5 is alternate interior to angle? ∠4

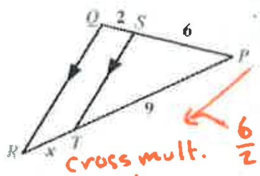
3) Angle 7 is corresponding to angle? ∠3

4) Angle 8 is vertical to angle? ∠5

5) If angle 1 equals 135 degrees, fill in all remaining angles.



6) Triangle Proportionality: Find x.

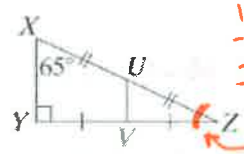


$\frac{6}{9} = \frac{2}{x}$ or $\frac{6}{2} = \frac{9}{x}$

cross mult.

$\frac{6x}{6} = \frac{18}{6}$ → $\boxed{x = 3}$

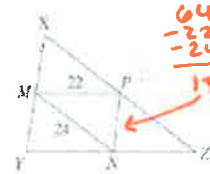
7) Similar Triangles: What is the reflexive angle? Is XZY similar to UZV? If so, how?



180
−90
−65
25

yes by AA~ or SAS~

8) Midsegment: If M, N, and P are midpoints & perimeter of MPN = 64, find the length of all segments.



64
−22
−24
18

$\overline{XZ} = 24 \cdot 2 = 48$

$\overline{XY} = 18 \cdot 2 = 36$

$\overline{YZ} = 22 \cdot 2 = 44$

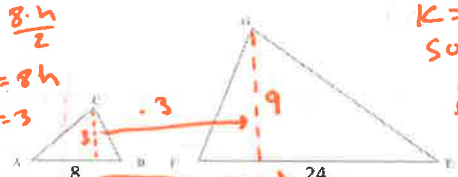
Perimeter = 128

9) The sketch below shows 2 similar Δ's, ABC and EFG. ABC has an area of 12 units, and it's base, AB, is 8 units long. The base of DEF is 24 units. What is the area of DEF?

$12 = \frac{8 \cdot h}{2}$

$24 = 8h$

$h = 3$



k = 3 so → $12 \cdot 3^2 = 108$ units²

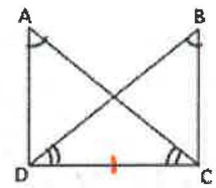
or find height of both

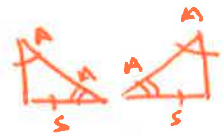
$A = \frac{b \cdot h}{2}$ → $A = \frac{24 \cdot 9}{2}$

$\boxed{A = 108}$

10) *Given:* ∡ *DAC* ≅ ∡*CBD*, ∡ *BDC* ≅ *ACD*

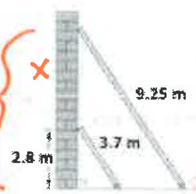
Prove: $\overline{AC} \cong \overline{BD}$





Statement	Reason
1. ∠DAC ≅ ∠CBD	1. Given
2. ∠BDC ≅ ∠ACD	2. Given
3. DC ≅ DC	3. Reflexive Property
4. Δ CDA ≅ Δ DCB	4. AAS
5. AC ≅ BD	5. CPCTC

11) What is the height between the tops of the two ladders?



$\frac{9.25}{3.7} = \frac{x + 2.8}{2.8}$

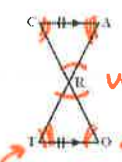
$25.9 = 3.7x + 10.36$
$-10.36 \qquad -10.36$

$\frac{3.7x}{3.7} = \frac{15.54}{3.7}$

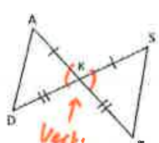
$\boxed{x = 4.2}$

Triangle congruency: ***SSS, SAS, ASA, AAS, HL.*** *Remember SSA / ASS can't prove congruency.* ***You can't double skip!***

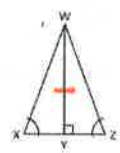
6) Δ *RAC* ≅ ΔRTO by AAS or ASA



7) Δ *KAD* ≅ ΔKST by SAS

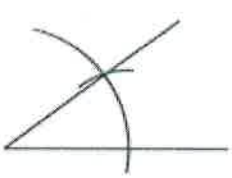


8) Δ *XYW* ≅ ΔZYW by AAS



Geometric Constructions

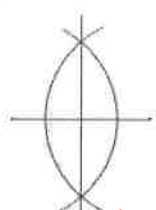
Copy Angle



Angle Bisector

Parallel Lines Pt

Perpendicular Bisector



Tangents

~~Rev~~ Review These!

Unit 3 - Right Triangle Trigonometry

Key Concepts

Finding missing sides use Soh Cah Toa

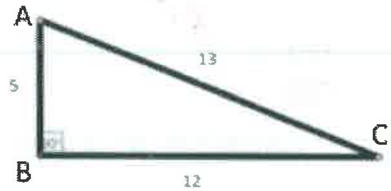
A missing side can be found using $\sin\theta = \left(\frac{o}{h}\right)$, $\cos\theta = \left(\frac{a}{h}\right)$, *or* $\tan\theta \left(\frac{o}{a}\right)$ *when you know an angle and one side of a right triangle.*

An angle θ *can be found by using one of* $\sin^{-1}\left(\frac{o}{h}\right)$, $\cos^{-1}\left(\frac{a}{h}\right)$, *or* $\tan^{-1}\left(\frac{o}{a}\right)$ *when two sides are known of a right triange.*

$Sin\ A = Cos\ B$ *when angles A and B are complementary in a right triangle:* $Sin\ A = Cos\ (90 - A)$

Using the diagram for 1-9. First, find each trig ratio.

1) $Sin\ A =$ $\frac{12}{13}$
2) $Cos\ A =$ $\frac{5}{13}$
3) $Tan\ A =$ $\frac{12}{5}$
4) $Sin\ C =$ 5/13
5) $Cos\ C =$ 12/13
6) $Tan\ C =$ 5/12



7) True or False: $Sin\ A = Cos\ (90 - A)$ Explain. True The sin of an angle is equal to the cos of its compliment

8) Tan A & Tan B are reciprocal

9) In a $45-45-90$ triangle, the ratio of Sin A = Cos A

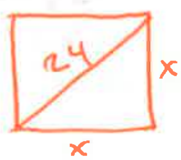
10) What is the length of the diagonal of a square with side lengths $3\sqrt{2}$? Pythagorean theorem or trig functions or special right triangles

$(3\sqrt{2})^2 + (3\sqrt{2})^2 = x^2$

$18 + 18 = x^2 \rightarrow x^2 = 36$

$\boxed{x=6}$

11) The length of the diagonal of a square is 24. What is the length of each side?



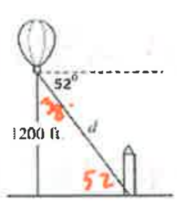
$x^2 + x^2 = 24^2$

$2x^2 = 576$

$x^2 = 288$

$\boxed{x = 16.97 \rightarrow 12\sqrt{2}}$

12) **Angle of Depression & Elevation**: If the AoD is 52 degrees, solve for d.



$\sin(52) = \frac{1200}{d}$

$d \cdot \sin(52) = 1200$

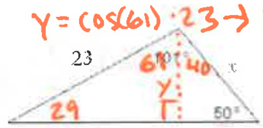
$d = \frac{1200}{\sin(52)}$

$\boxed{d = 1522.82ft}$

13) **Drop an altitude**: Solve for x.

$\cos(61) = \frac{y}{23}$

$y = \cos(61) \cdot 23 \rightarrow 11.15$



$\sin(50) = \frac{11.15}{x}$

$x = 11.15 \cdot \sin(50) \rightarrow \boxed{8.54}$

14) **Area:** Solve for total area.

$\tan(50) = \frac{41}{x}$

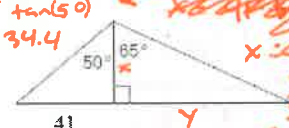
$x = \frac{41}{\tan(50)}$

$x = 34.4$

$\tan(65) = \frac{y}{34.4}$

$34.4 \cdot \tan(65)$

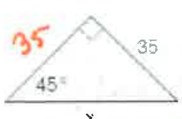
$y = 73.78$



114.28

$\boxed{1974.22}$

15) **Regular Trig:** Find the missing side.



$35 \cdot \sqrt{2} \rightarrow \boxed{49.5}$

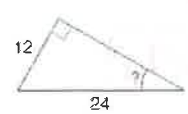
16) **Regular Trig:** Find the missing side.

$\cos(73) = \frac{x}{6}$

$x = 6 \cdot \cos(73)$

$\boxed{x = 1.75}$

17) **Inverse Trig:** Find the angle.

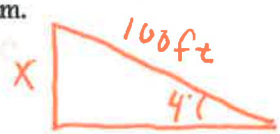


$\sin(x) = \frac{12}{24}$

$\sin^{-1}\left(\frac{12}{24}\right) = x$

$\boxed{x = 30°}$

18) A road ascends a hill at an angle of 4° for every 100 feet of road, how many feet does the road ascend? **Draw a diagram.**

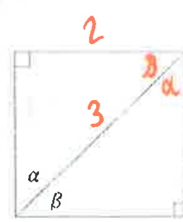


$\sin(4) = \frac{x}{100} \rightarrow 100 \cdot \sin(4) = x$

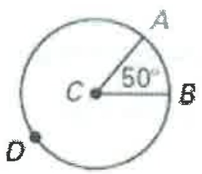
$x = 6.98ft$

19) In this figure, two right angles and two adjacent angles, α & β, are shown. If $\sin(\alpha) = \frac{2}{3}$, what is the value of $\cos(\beta)$?

$\cos(\beta) = \frac{2}{3}$

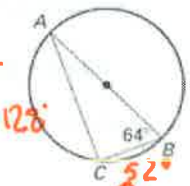


Central ∡ = Intercepted arc



AB = ? 50°

Diameter of Inscribed ∡ $m\widehat{BC}$



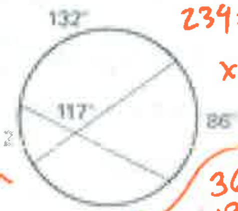
64.2 → 128°

52°

180
−128

Interior ∡ = ½ $\frac{B+L}{2}$

Find arcs 1 & 2.



$117 = \frac{132+x}{2}$

$234 = 132 + x$

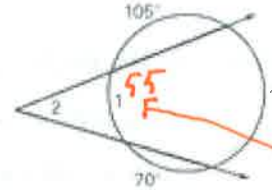
$x = 102$

360
−132
−102
−86

x

40

Exterior ∡ = ½ $\frac{B-L}{2}$



55

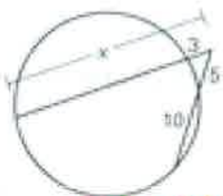
360
−105
−130
−70

Find arc 1 & angle 2/

$\angle 2 = \frac{130-55}{2}$

$\boxed{\angle 2 = 37.5°}$

Outside(whole) =

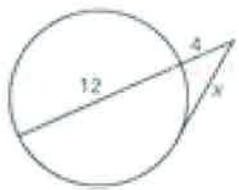


Find x. $3(x) = 5(15)$

$3x = 75$

$\boxed{x = 25}$

Tangent is outside & whole

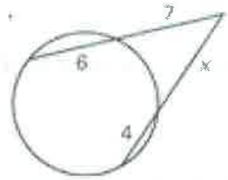


Find x. $4(16) = x(x)$

$64 = x^2$

$\boxed{x = 8}$

Missing Outside = Quadratic



Find x. $x(x+4) = 7(13)$

$x^2 + 4x = 91$

$x^2 + 4x - 91 = 0$ $x = 7.75$

use quad form

Shared Inscribed ∡'s are ≅

$2x + 8 = 3x - 24$

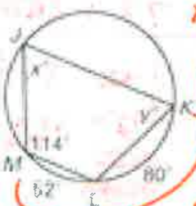
$\boxed{32 = x}$

$2(32) + 8$

$\boxed{\angle BCA = 72°}$

Find x & angle BCA.

Inscribed Quad is Supplementary



$x = \frac{132}{2}$

$\boxed{x = 66°}$

$y + 114 = 180$

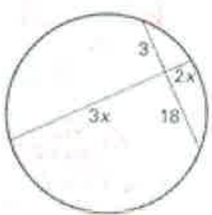
$\boxed{y = 66}$

132

Find x & y.

Pay attention to arc MLK.

Product of Parts



Find x and lengths of chords.

$3(18) = 3x(2x)$ → $x = 3$

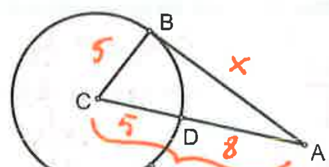
$54 = 6x^2$

$9 = x^2$

Chords → 21 & 17

Point of Tangency: AB is tangent to circle C at B.

AD=8, CB=5, AB = ?



$13^2 - 5^2 = x^2$

$144 = x^2$

$\boxed{x = 12}$

Arc Length & Sector Area

An apple pie has a diameter of 9 in. The pie is cut into 6 equal pieces. What is the area and arc length of 4 pieces of pie?

Area

$\frac{4}{6} \cdot \pi \cdot 4.5^2 \to \frac{27}{2}\pi \to \boxed{42.41 \text{ in}^2}$

length

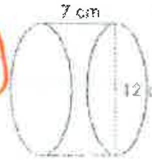
$\frac{4}{6} \cdot 2 \cdot \pi \cdot 4.5 \to 6\pi \to \boxed{18.85 \text{ in}}$

Sphere Volume:

$\frac{4}{3}\pi\, 6.4^3$

$\boxed{V = 1098.07 \text{ yd}^3}$

Cylinder Volume:

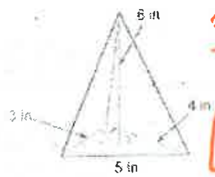


$\pi \cdot 6^2 \cdot 7$

$V = 252\pi$

$\boxed{V = 791.68}$

RT Triangle Pyramid volume:



$\frac{3 \cdot 4 \cdot 6}{6}$

$\boxed{12 \text{ in}^3}$

Cavalieri's Principle: Can the cylinder and RTΔ pyramid at left have the same volume if they have the same height? Explain. No only if cross sections have equal areas

Cone Volume:

$\frac{1}{3}\pi 12^2 \cdot 24$

1152π

$\boxed{V = 3619.11 \text{ m}^3}$

Composite Volume:

$V_{cyl} = \pi \cdot 6^2 \cdot 13$

468π

$V_{sp} = \frac{4}{3}\pi \cdot 6^3 = 288\pi$

$\frac{288\pi}{2}$

144π

$468\pi + 144\pi$

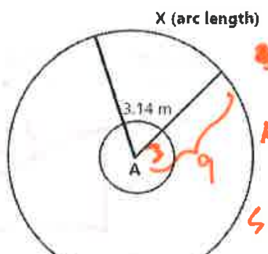
$\boxed{1922.65}$

At right, the radius of the smaller circle = 3 m while the radius of the larger circle is 9 m. The arc length intercepted by the small circle is 3.14 m. what is the arc length of the larger circle? s = AL, r = radius

$\frac{s}{r} = \frac{s}{r}$ $\frac{3.14}{3} = \frac{x}{9}$

$28.26 = 3x$

$\boxed{x = 9.42 \text{m}}$



3

9

$K = \frac{9}{3} = 3$

60°

or $3.14 \cdot 3 =$

$\boxed{9.42 \text{m}}$

Identify 2D shapes as 3D Objects: If a circle is rotated, what 3D shape will results?

Sphere

Unit 5 – Algebraic Connections with Geometry

Key Concepts

Distance: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, and you can always draw a right triangle on a graph to find Δx *and* Δy.

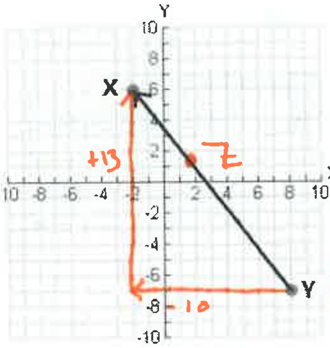
Midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Point Partitioning a Line Segment: $(x, y) = (x_1 + \frac{A}{A+B}(\Delta x), y_1 + \frac{A}{A+B}(\Delta y))$

Standard Form of a Circle: $(x-h)^2 + (y-h)^2 = r^2$, **where the number on the right is ALWAYS squared.**

A parallelogram and rhombus have diagonals that bisect. A rectangle and square have diagonals that are congruent.

1) **Partitioning:** Find Point Z that partitions the directed line segment $\overline{YX}$ in a ratio of $\frac{5}{3}$, $X(-2, 6)$ *and* $Y(8, -7)$. Graph.



$\frac{5}{3} \rightarrow \frac{5 \text{ Parts}}{8 \text{ Parts}}$

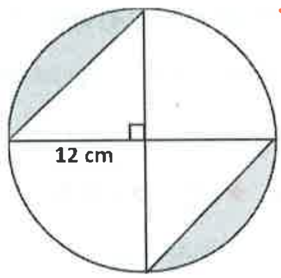
$\frac{5}{8}\left(\overset{\Delta x}{-10}, \overset{\Delta y}{13}\right)$

$[-6.25, +8.125]$

$(8 - 6.25, -7 + 8.125$

$\boxed{Z = (1.75, 1.125)}$

2) **Sector Area:** 2 diagonals of a circle are shown, and the radius is 12 cm. What is the area of the shaded regions?



Sector Area =

$\frac{90}{360} \cdot \pi \cdot 12^2 \rightarrow 36\pi$

Triangle Area =

$\frac{12 \cdot 12}{2} \rightarrow 72$

Shaded Area

$36\pi - 72 \rightarrow 41.1$

times 2

$\boxed{82.2 \text{ cm}^2}$

3) **Completing the Square:** Put into standard form, find center & radius. $4x^2 + 4y^2 - 24x + 48y + 13 = 0$

4

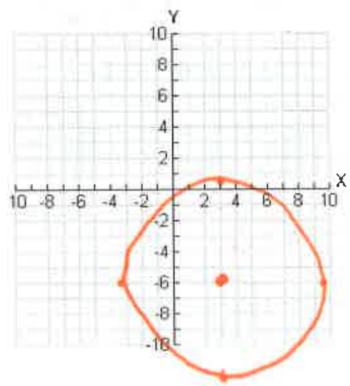
$x^2 - 6x + \_\_ + y^2 + 12y + \_\_ = -3.25 + \_\_ + \_\_$

$\frac{-6}{2} = -3^2 = 9$ $\frac{12}{2} = 6^2 = 36$

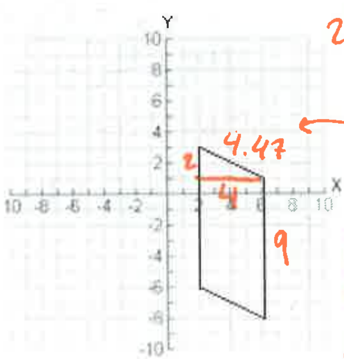
$(x-3)^2 + (y+6)^2 = 41.75$

$C = (3, -6)$ $r = \sqrt{41.75} = 6.46$

4) **Graphing Circles:** Now graph the circle from #3.



5) **Distance Formula:** Find the perimeter and area.



$2^2 + 4^2 = x^2$

$20 = x^2$

$x = 4.47$

$P = 2 \cdot 9 + 2 \cdot 4.47$

$\boxed{P = 26.94}$

$A = 9 \cdot 4$

$\boxed{A = 36}$

6) **Circle Properties:** Which point shown below lies on a circle with a center of $(3, -9)$ and a radius of $\sqrt{34}$?

$(6, -3)$ or $(1, -2)$ or $(1, -4)$ or $(0, -4)$

write equation for circle. Plug in points to see what works $\longrightarrow (x-3)^2 + (y+9)^2 = 34$

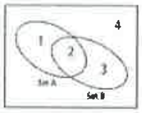
7) Find the midpoint: $(-10, -5)$ & $(13, 8)$

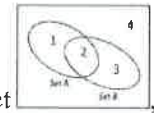
$mp = \left(\frac{-10+13}{2}, \frac{-5+8}{2}\right)$

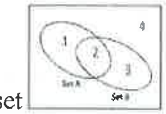
$\boxed{mp = (1.5, 1.5)}$

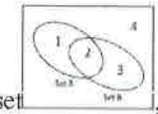
Unit 6 - Probability

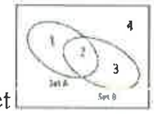
Key Concepts

Given $A \cup B$ shade the set 

, *Given* $A \cap B$ shade the set 

, *Given* $\bar{A}$ *or* A' shade the set 

Given $(\bar{A} \cup \bar{B})'$ shade the set 

, *Given* $(A \cap B)'$ shade the set 

Addition Rule (aka mutually exclusive): $\boldsymbol{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

Multiplication Rule for Independent Events: $\boldsymbol{P(A \cap B) = P(A) * P(B)}$

Conditional Probability: $\boldsymbol{P(A \cap B) = P(A) * P(B/A)}$ **or** $\boldsymbol{P(B|A) = \frac{P(A \cap B)}{P(A)}}$

Independent Events do not affect one another while Dependent Events do and means non-replacement.

Car Ownership by Grade

	Owns a Car	Does Not Own a Car
Junior	6	10
Senior	12	8
TOTAL	18	18

1) Find the probability that a randomly selected student will be a junior, given that the student owns a car.

$P(J|oc) \rightarrow \frac{6}{18} \rightarrow \frac{1}{3} \rightarrow .33$

2) Find the probability that a randomly selected student will own a car, given that the student is a senior.

$P(oc|S) \rightarrow \frac{12}{20} \rightarrow \frac{3}{5} \rightarrow .6$

3) For two events B and C, it is known that $P(C|B) = 0.65$ and $P(C \cap B) = .43$. Find P(B).

$P(C|B) = \frac{P(C \cap B)}{P(B)} \rightarrow .65 = \frac{.43}{x} \rightarrow$ $.65x = .43$ $\boxed{x = .66}$

4) For two events X and Y, it is known that $P(X) = \frac{5}{24}$ and $P(X \cap Y) = \frac{1}{8}$. Find $P(Y|X)$.

$P(Y|X) = \frac{P(X \cap Y)}{P(X)} \rightarrow x = \frac{1/8}{5/24} \rightarrow \frac{1}{8} \cdot \frac{24}{5} = \boxed{\frac{3}{5} = .60}$

4) A sock drawer contains 5 pairs of each color socks: white, green and blue. What is the probability of randomly selecting a pair of blue socks, replacing it, and then randomly selecting a pair of white socks?

P(B) and P(B)

$\frac{5}{15} \cdot \frac{5}{15} = \frac{25}{225} = \frac{1}{9} = .\overline{11}$

5) Randy has 8 pennies, 3 nickels, and 5 dimes in his pocket. If he randomly chooses 2 coins, what is the probability that they are both pennies if he doesn't replace the first one?

P(P) • P(P)

$\frac{8}{16} \cdot \frac{7}{15} = \frac{56}{240} = \frac{7}{30} = .2\overline{3}$

6) Using the letters in the state MISSISSIPPI. Find the probability of picking an **S** and then a **P** without replacement.

$\frac{4}{11} \cdot \frac{2}{10} = \frac{8}{110} = \frac{4}{55} = .0\overline{72}$

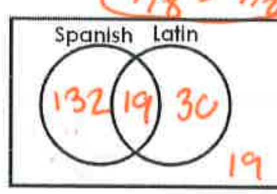
7) Using $\boldsymbol{P(A \cap B) = P(A) * P(B)}$, determine if the following events are independent.

$\boldsymbol{P(A) = \frac{3}{4}, P(B) = \frac{5}{6}, P(A \cap B) = \frac{5}{8}}$.

$\frac{3}{4} \cdot \frac{5}{6} = \frac{5}{8}$ So yes!

$5/8 = 5/8$

A guidance counselor is planning schedules for 200 students. 151 want to take Spanish and 49 want to take Latin. 19 say they want to take both. Display this information on the Venn Diagram.



8) What's the probability that a student studies at least one subject? P(SL) $\frac{181}{200} = .91$

9) What's the probability that a student studies exactly one subject? $\frac{162}{200} = .81$

10) What's the probability that a student studies neither subject? P(SL)' $\frac{19}{200} = .10$

11) What's the probability that a student studied Spanish if it is known that he, she studies Latin? $\frac{19}{49} = .39$

12) If you roll two die, find:

P(Odd number or a number greater than 8)

$\underset{\text{odds}}{\frac{18}{36}} + \underset{>8}{\frac{15}{36}} - \underset{\text{overlap}}{\frac{6}{36}} \rightarrow \boxed{\frac{27}{36} = .75}$

13) If you roll two die, find:

P(Doubles or a sum of 6)

$\frac{6}{36} + \frac{5}{36} - \frac{1}{36} \rightarrow \frac{10}{36} = .2\overline{7}$