

Unit 5 Study Guide

Name Key \_\_\_\_\_ S \_\_\_\_\_

- Which information is needed to show that a parallelogram is a rectangle?
  - The diagonals bisect each other.
  - The diagonals are congruent.
  - The diagonals are congruent and perpendicular.
  - The diagonals bisect each other and are perpendicular.

- Using A-D from #1, which information is needed to prove a parallelogram?
 

A. diagonals bisect

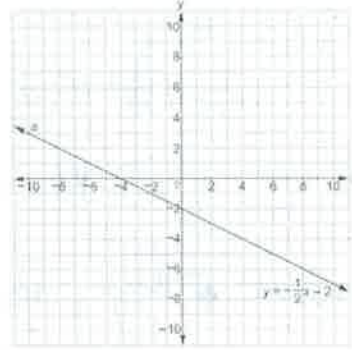
- Given the points  $P(2, -1)$  &  $Q(-9, -6)$ , what are the coordinates of the point on the directed line segment  $\overline{PQ}$  that partitions  $\overline{PQ}$  into the ratio  $\frac{3}{2}$ ?

A.  $(-\frac{23}{5}, -4)$   
 B.  $(-\frac{12}{5}, -3)$   
 C.  $(\frac{5}{3}, \frac{8}{3})$   
 D.  $(-\frac{5}{3}, -\frac{8}{3})$

$\frac{3}{5}(-11, -5)$   
 $[2 - \frac{33}{5}, -1 - 3]$   
 $(-\frac{23}{5}, -4)$

$x_1$        $y_1$

- An equation of a line  $a$  is  $y = -\frac{1}{2}x - 2$ . See graph.



What is the equation of the line that is perpendicular to line  $a$  shown on the graph and passes through point  $(-4, 0)$ .

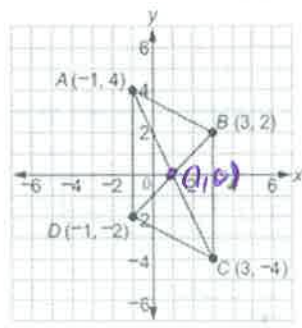
- Perp means slope is opp. recip.
- $m = -\frac{1}{2} \rightarrow m = \frac{2}{1}$
- $y = -\frac{1}{2}x + 2$
  - $y = -\frac{1}{2}x + 8$
  - $y = 2x - 2$
  - $y = 2x + 8$
- Plug in  $x$  and  $y$  to see what equation works

- Which point is on a circle with a center of  $(3, -9)$  and a radius of 5?

A.  $(-6, 5)$   
 B.  $(-1, 6)$   
 C.  $(1, 6)$   
 D.  $(6, -5)$

$(x-3)^2 + (y+9)^2 = 25$   
 Plug in to see what works

- Parallelogram ABCD has vertices as shown.



Write out the two sets, AC & BD, of the full distance formulas set equal to each other that would be used to prove that the diagonals of ABCD bisect each other? Then solve.

$AC \rightarrow \sqrt{(-1-1)^2 + (4-6)^2} = 2\sqrt{5} = 4.47$   
 $\sqrt{(1-3)^2 + (0+4)^2} = 2\sqrt{5} = 4.47$

$DB \rightarrow \sqrt{(3-1)^2 + (2-0)^2} = 2\sqrt{2} = 2.83$   
 $\sqrt{(1+1)^2 + (0+2)^2} = 2\sqrt{2} = 2.83$

Use the information provided to write the standard form of a circle.

7. Center:  $(2\sqrt{3}, -5\sqrt{2})$ , Radius =  $\sqrt{13}$

$$(x - 2\sqrt{3})^2 + (y + 5\sqrt{2})^2 = 13$$

8. Center:  $(4, -14)$  and the point  $(6, 11)$  that lies on the circle.

$$d = \sqrt{(6-4)^2 + (11+14)^2} = \sqrt{629}$$

$$r = \frac{d}{2} = \frac{\sqrt{629}}{2}$$

$$(x-4)^2 + (y+14)^2 = \frac{629}{4}$$

Use the information provided to write the general conic form of a circle.

9.  $(x+10)^2 + (y-7)^2 = 9$

$$(x+10)(x+10) + (y-7)(y-7) - 9 = 0$$

$$x^2 + 20x + 100 + y^2 - 14y + 49 - 9 = 0$$

$$x^2 + y^2 + 20x - 14y + 140 = 0$$

10.  $(x-14)^2 + (y+14)^2 = 9$

$$(x-14)(x-14) + (y+14)(y+14) - 9 = 0$$

$$x^2 - 28x + 196 + y^2 + 28y + 196 - 9 = 0$$

$$x^2 + y^2 - 28x + 28y + 383 = 0$$

Use the information provided to write the standard form of a circle. Then identify the center and radius length.

11.  $x^2 + y^2 - 20x + 2y + 76 = 0$

$$x^2 - 20x + \frac{100}{2} + y^2 + 2y + \frac{1}{2} = -76 + \frac{100}{2} + \frac{1}{2}$$

$$\frac{-20}{2} = -10^2 = 100 \quad \frac{2}{2} = 1^2 = 1$$

$$(x-10)^2 + (y+1)^2 = 25$$

$C = (10, -1) \quad r = 5$

12.  $2x^2 + 2y^2 + 28x + 24y + 21 = 0$

$$x^2 + y^2 + 14x + 12y + 10.5 = 0$$

$$x^2 + 14x + \frac{49}{2} + y^2 + 12y + \frac{36}{2} = -10.5 + \frac{49}{2} + \frac{36}{2}$$

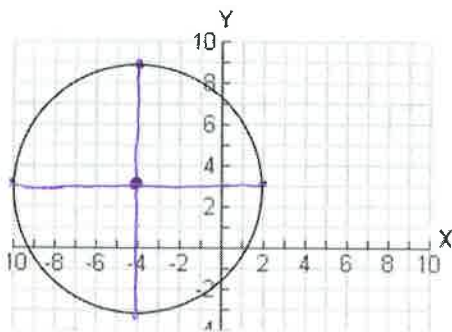
$$\frac{14}{2} = 7^2 = 49 \quad \frac{12}{2} = 6^2 = 36$$

$$(x+7)^2 + (y+6)^2 = 74.5$$

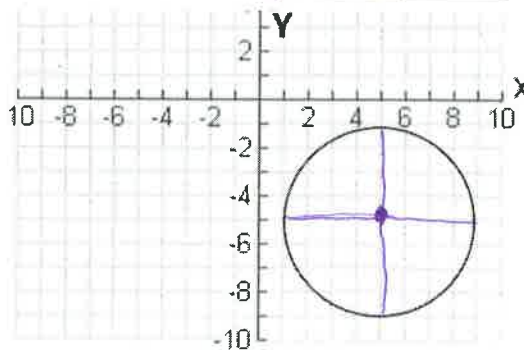
$C = (-7, -6) \quad r = 8.63$

Find the center and the radius length to write the standard form of each circle.

13.  $(x+4)^2 + (y-3)^2 = 36$



14.  $(x-5)^2 + (y+5)^2 = 16$



15. Prove or disprove that the points  $A(8, 6)$ ,  $B(8, -6)$  and  $C(-10, 0)$  are the vertices of an isosceles triangle inscribed in the circle centered at the origin  $O$  and passing through the point  $P(3, \sqrt{91})$ .

$$\begin{aligned} \overline{AB} &= \sqrt{(8-8)^2 + (-6-6)^2} = 12 \\ \overline{BC} &= \sqrt{(-10-8)^2 + (0+6)^2} = 18.97 \\ \overline{AC} &= \sqrt{(-10-8)^2 + (0-6)^2} = 18.97 \end{aligned} \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{same so } \Delta \text{ is isos.}$$

$$\begin{aligned} d_P &= \sqrt{(3-0)^2 + (\sqrt{91}-0)^2} = \sqrt{100} = 10 \\ d_A &= \sqrt{(8-0)^2 + (6-0)^2} = \sqrt{100} = 10 \\ d_B &= \sqrt{(8-0)^2 + (-6-0)^2} = \sqrt{100} = 10 \\ d_C &= \sqrt{(-10-0)^2 + (0-0)^2} = \sqrt{100} = 10 \end{aligned} \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \text{All same so it is}$$

On a coordinate plane, a local television station is located at the origin and has a broadcast range of 50 miles.

- 16) Write an equation that represents the region covered by this television station.

$$C=00 \quad r=50$$

$$\boxed{x^2 + y^2 = 2500}$$

- 17) Can a person who lives 18 miles to the East and 35 miles North of the station watch this TV station?

distance surely must be 50 or less!

$$d = \sqrt{(18-0)^2 + (35-0)^2} = \sqrt{1549}$$

$$\boxed{39.36 \text{ mi.} \rightarrow \text{yes!}}$$

You're a city planner, so you know that streets run north to south and avenues run east to west. Your friend Melissa lives at the corner of 3rd Street and 28th Avenue. Her sister Rebecca lives at the corner of 27th Street and 16th Avenue. If necessary, draw a graph to find the cross street that meets each criteria.

18. Is  $\frac{2}{3}$  of the way from Melissa's to Rebecca's.

$$\frac{2}{3} \left( \begin{array}{l} +24, -12 \\ \downarrow \quad \downarrow \\ [3+16, 28-8] \end{array} \right)$$

$$\boxed{(19, 20)}$$

19. From Melissa's home to Rebecca's home by a  $\frac{1}{5}$  ratio.

$$\frac{1}{5} \left( \begin{array}{l} 24, -12 \\ \downarrow \quad \downarrow \\ [3+4, 28-2] \end{array} \right)$$

$$\boxed{(7, 26)}$$

Determine if point A lies on a circle with center C and point P which is known to lie on the circle.  
create equation and plug in or check distance

20.  $A(5, 0)$ ,  $C(0, 0)$ ,  $P(3, 4)$

$$\overline{AC} = \sqrt{(5-0)^2 + (0-0)^2} = \sqrt{25} = 5$$

$$\overline{CP} = \sqrt{(3-0)^2 + (4-0)^2} = \sqrt{25} = 5$$

$$\boxed{\text{equal distance so yes!!}}$$

21.  $A(0, 4)$ ,  $C(2, 1)$ ,  $P(5, 3)$

$$\overline{AC} = \sqrt{(0-2)^2 + (4-1)^2} = \sqrt{13}$$

$$\overline{CP} = \sqrt{(2-5)^2 + (1-3)^2} = \sqrt{13}$$

$$\boxed{\text{equal so yes!!}}$$

- 22) When proving a rectangle is a parallelogram, which method is the best choice?

Show that the diagonals bisect by finding the midpoint of both diagonals.

Show that the diagonals have the same length using the distance formula or the Pythagorean Formula.

- 23) When proving the parent parallelogram, which method is the best choice?

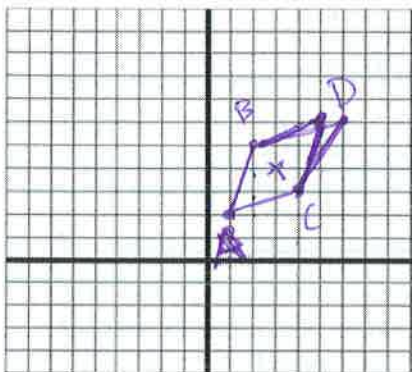
Show that the diagonals bisect by finding the midpoint of both diagonals.

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For each figure using, prove the type of quadrilateral, using distance and, or slope. Keep diagonals in mind.

24. ABCD: A(1, 2), B(2, 5), C(4, 3), D(5, 6)



find midpoint of AD & BC

$$MP \cdot AD = \left(\frac{1+5}{2}, \frac{2+6}{2}\right) = (3, 4) \checkmark$$

$$MP \cdot BC = \left(\frac{2+4}{2}, \frac{5+3}{2}\right) = (3, 4) \checkmark$$

Parallelogram

Write the equation of the lines below in slope-intercept form:  $y = mx + b$ .

26. Through  $(-4, 5)$  and parallel to  $y = -\frac{3}{2}x - 5$ .

$$m = -\frac{3}{2} \rightarrow m = -\frac{3}{2}$$

$$y = -\frac{3}{2}x + b$$

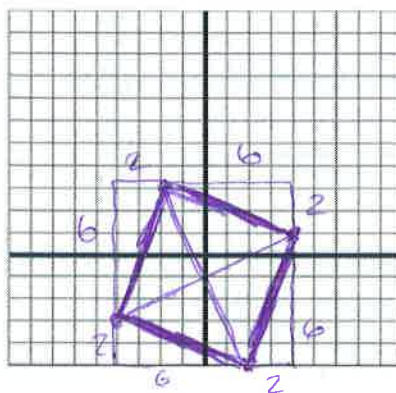
$$5 = -\frac{3}{2}(4) + b$$

$$5 = -6 + b$$

$$b = 1$$

$$y = -\frac{3}{2}x + 1$$

25. EFGH: E(4, 1), F(-2, 3), G(2, -5), H(-4, -3)



sides are Perp. and  $\cong$  so square.

diagonals are  $\cong$  and perp so square.

$$\sqrt{(6)^2 + (2)^2} = 6.32 \checkmark$$

$$\sqrt{(2)^2 + (6)^2} = 6.32 \checkmark$$

$$-\frac{6}{2} \leftrightarrow \frac{2}{6}$$

27. Through  $(4, 1)$  and perpendicular to  $y = -2x - 2$

$$m = -2 \rightarrow m = \frac{1}{2}$$

$$y = \frac{1}{2}x + b$$

$$1 = \frac{1}{2}(4) + b$$

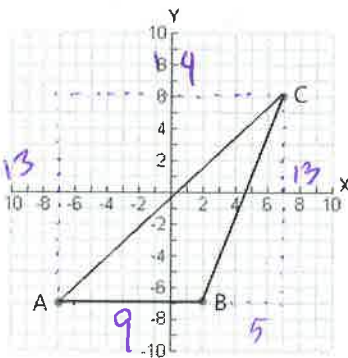
$$1 = 2 + b$$

$$b = -1$$

$$y = \frac{1}{2}x - 1$$

Find the area and perimeter of the following triangle. Simplest form required. Reminder: Draw altitude to find height.

28. Area = 58.5 29. Perimeter = 42.03



$$A = \frac{b \cdot h}{2} = \frac{9 \cdot 13}{2}$$

$$P = \overline{AB} = 13$$

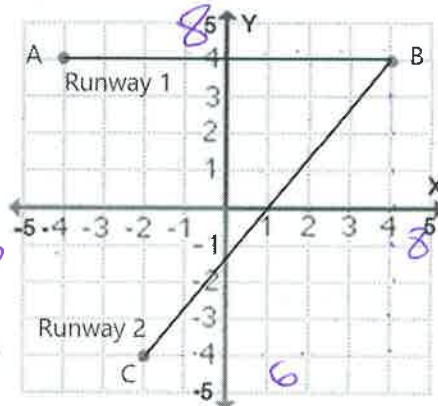
$$\overline{BC} = \sqrt{5^2 + 13^2} = 13.93$$

$$\overline{AC} = \sqrt{13^2 + 14^2} = 19.1$$

sum together!

In the diagram, two runways intersect at point B. Each square is 200 x 200 yards square. If you walked from A to B and then to C, how far did you walk?

30) 18 yards



$$\overline{BC} = \sqrt{6^2 + 8^2}$$

$$\downarrow$$

$$\sqrt{100}$$

$$\downarrow$$

$$10$$

$$10 + 8$$